Spectral Dimension of kappa space-time

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Contents

Introduction

- Recent studies of space-time structure at high energies show the possibility of a dimensional flow.
- The effective dimension of a quantum space-time need not be same as the topological dimension.
- At high energy scale, space-time appears to be fuzzy due to quantum effects. The blurriness of the space-time screens some dimension, reducing the number of accessible dimensions.
- This gives us a hope for a renormalizable theory of gravity.

What is spectral dimension?

- One can use diffusion process to measure the effective dimension, which is termed as spectral dimension.
- Intuitively : Corresponds to the dimension that a random walker 'experiences' in a diffusion process.
- Spectral dimension is known to change as one probes at high energies.

Concepts of Spectral dimension

- **Diffusion equation**

$$\frac{\partial}{\partial \sigma} U(x, y; \sigma) = \mathcal{L} U(x, y; \sigma)$$

  - σ is a fictitious diffusion time.
  - $\mathcal{L}$ is the Laplace operator.
  - U(x,y;σ) is the heat kernel representing the probability density of diffusion from x to y in diffusion time σ.

- **Return probability**

$$P_g(\sigma) = \frac{\int d^n x \sqrt{detg_{\mu\nu}} U(x, x; \sigma)}{\int d^n x \sqrt{detg_{\mu\nu}}}$$

- Logarithmic derivative of $P_g(\sigma)$ will give the **spectral dimension**.

$$D_s = -2 \frac{\partial \ln P(\sigma)}{\partial \ln \sigma}$$

Classical flat space-time

- Diffusion equation associated with a n-dimensional Euclidean geometry

$$\frac{\partial}{\partial\sigma}U(x,y;\sigma) = \nabla^2 U(x,y;\sigma)$$

  Initial condition

$$U(x,y;0) = \delta^n(x-y)$$

- Heat kernel

$$U(x,y;\sigma) = \frac{1}{(4\pi\sigma)^{\frac{n}{2}}} e^{\frac{-(x-y)^2}{4\sigma}}$$

- **Return probability**

$$P_g(\sigma) = \frac{\int d^n x \sqrt{det g_{\mu\nu}} U(x, x; \sigma)}{\int d^n x \sqrt{det g_{\mu\nu}}}$$

$$P_g(\sigma) = (4\pi\sigma)^{-\frac{n}{2}}$$

- **Spectral dimension**

$$D_s = n$$

  - The spectral dimension is same as that of topological dimension n.
  - It is independent of σ.

Non-commutative space-time

- Concept of non-commutative (NC) space-time was introduced by H. Snyder to deal with the UV divergence in QFT.
- Different approaches to study the microscopic structure of space-time, introduced the notion of a minimum length scale(S. Doplicher, K. Fredenhagen, J. E. Roberts; Comm. math. phys. **172**,1995).
- Space-time points will be replaced by cell and points become fuzzy.
- NC is a possible way to capture the structure of space-time at microscopic level.
- In recent times, NC space-time were brought back by developments in string theory and quantum gravity.

Kappa deformed Euclidean space

- Coordinates obey Lie-Algebraic type of commutation relation.

$$[\hat{x}_n, \hat{x}_i] = ia\hat{x}_i \qquad [\hat{x}_i, \hat{x}_j] = 0$$

  where a is the deformation parameter.
- Two approaches:
  - Work with non-commutative functions defined on NC space-time.
  - Map function of NC coordinate to function on commutative space-time and use these non-local functions to setup the models (S. Meljanac and M. Stojic, Eur. Phys. J. C47 (2006) 531).

- Realization of the NC coordinates can be obtained in terms of commutative coordinates and its derivative by demanding linearity in x_μ and ∂_μ.

$$\begin{aligned}\hat{x}_i &= x_i \varphi(A) \\ \hat{x}_n &= x_n \psi(A) + i a x_k \partial_k \gamma(A)\end{aligned}$$

  where $A = ia\partial_n$

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$$[M_{\mu\nu}, M_{\lambda\rho}] = \delta_{\nu\lambda} M_{\mu\rho} - \delta_{\mu\lambda} M_{\nu\rho} - \delta_{\nu\rho} M_{\mu\lambda} + \delta_{\mu\rho} M_{\nu\lambda}$$

- Undeformed Poincare algebra with modified generators.

- Derivatives D_μ which transform as a 4-vector under κ-undeformed Poincare algebra

$$D_i = \partial_i \frac{e^{-A}}{\varphi}, \quad D_n = \partial_n \frac{sinhA}{A} + ia\nabla^2 \frac{e^{-A}}{2\varphi^2}$$

- Casimir of the undeformed κ-poincare algebra

$$D_\mu D_\mu = \Box(1 - \tfrac{a^2}{4}\Box)$$
$$\Box = \nabla^2 \tfrac{e^{-A}}{\varphi^2} - \partial_n^2 \tfrac{2(1-coshA)}{A^2}$$

- Expand up to first non-vanishing terms in ‘a’

$$D_\mu D_\mu = \nabla^2 + \partial_n^2 - \frac{a^2}{3}\partial_n^4 - \frac{a^2}{2}\nabla^2\partial_n^2 - \frac{a^2}{4}\nabla^4$$

Spectral dimension

- Modified diffusion equation

$$\frac{\partial U}{\partial \sigma} = \Box(1 - \frac{a^2}{4}\Box)U$$

- First non vanishing terms in ‘a’

$$\frac{\partial U}{\partial \sigma} = \nabla_n^2 U - \frac{a^2}{3}\partial_n^4 U - \frac{a^2}{2}\nabla^2\partial_n^2 U - \frac{a^2}{4}\nabla^4 U$$

- Perturbative solution

$$U = U_0 + aU_1 + a^2 U_2$$

- The return probability

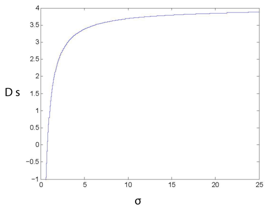
$$P_g(\sigma) = \frac{1}{(4\pi\sigma)^{\frac{n}{2}}}[1 + a\alpha + a^2\beta - a^2\frac{(n+1)^2}{16\sigma}]$$

- **Spectral dimension**

$$D_s = n - (n+1)^2\frac{a^2}{8\sigma}$$

D_s vs σ

$a = 1 \qquad n = 4$



Conclusion

- For classical flat space-time the spectral dimension is same as the topological dimension.

- In case of NC space-time spectral dimension shows length scale dependence.

  For a four dimensional space-time

  - $\sigma \gg a^2$ $\quad D_s \sim 4$
  - $\sigma > a^2$ $\quad D_s < 4$
  - $\sigma \simeq a^2$ $\quad D_s \simeq .875$
  - $\sigma \ll a^2$ $\quad D_s < 0$

References

- J. Ambjorn, J. Jurkiewicz and R. Roll, Phys. Rev. Lett. 95(2005)171301
- L. Modesto, P. Nicolini, Phys. Rev. D81(2010).104040
- S. Meljanac, Marko Stojic, arXiv:hep-th/0605133v2
- M. Arzano, T. Trzesniewski, arXiv: 1404.4762v1
- G. Calcagni, G. Nardelli, Phys. Rev. D 88, 124025(2013)

Thank You...