Backreaction effects of matter coupled higher derivative gravity

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(Based on arXiv:1409.8019, work done with Ramadevi)

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Introduction

Strongly coupled field theories

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- **Specific limit:**

Fluid-Gravity Correspondence

Hydrodynamics: Large length scales in field theories.

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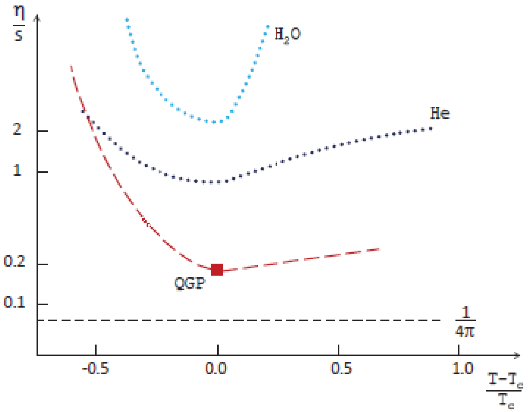

Figure: arXiv:1206.3581

- **The figure shows expectation taking analogy with H_2O and He plots.**

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Use AdS-CFT correspondence

$$\langle e^{i\int_{\partial M}\phi_0\tilde{O}}\rangle = e^{iS_{cl}[\phi]}$$

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Giving,

$$\langle \tilde{O}(x_1)\tilde{O}(x_2)\rangle = i^2 \frac{\delta^2}{\delta\phi_0(x_1)\delta\phi_0(x_2)} e^{iS_{cl}[\phi_0]} \qquad \textbf{at } \phi_0 \to 0$$

Note: We need to know the background geometry for the above calculations.

Background solutions in gravity

Einstein gravity with higher derivatives

$$S = \int d^D x \sqrt{-g} \left(R - 2\Lambda + c_1 R^2 + c_2 R^{\mu\nu} R_{\mu\nu} + c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \right) \quad (1)$$

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with
$$\Lambda = -\frac{(D-1)(D-2)}{2} \quad (2)$$
$$ds^2 = \frac{1}{r^2}\left(\left(-f(r)dt^2 + (d\vec{x}^2)\right) + \frac{dr^2}{f(r)} \right) \quad (3)$$

where, $f(r) = 1 - r^{D-1} + \delta + \kappa r^{2(D-1)}$

δ and κ being dependent on D, c_1, c_2 and c_3
(arXiv: 0712.0743)

Matter coupling

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Consider the action

$$S = \frac{1}{2\kappa_D^2}\int d^D x[R - \frac{4}{D-2}(\partial\Phi(r))^2 - V(\Phi(r))] \ ; \ V(\Phi(r)) = 2\Lambda e^{\alpha\Phi(r)} \tag{4}$$

The already known solution to this action shows effects of matter on the background metric

It is interesting to look upon the case of matter coupled higher derivative gravity

$$S = \frac{1}{2\kappa_D^2} \int d^D x (R - \frac{4}{D-2}(\partial\Phi(r))^2 - V(\Phi(r)) + \beta G(\Phi(r)) R_{\mu\nu\sigma\rho} R^{\mu\nu\sigma\rho}) \tag{5}$$

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Varying the action with respect to metric field and scalar field gives,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}^{matter} + T_{\mu\nu}^{hd} \tag{6}$$

$$\frac{8}{D-2}\Box\Phi(r) = \partial_\Phi\big(\beta G(\Phi(r))R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} - V(\Phi(r))\big) \tag{7}$$

$$\text{where}, T_{\mu\nu} = -\frac{1}{\sqrt{-g}}\frac{\partial\sqrt{-g}L_M}{\partial g^{\mu\nu}} \tag{8}$$

- **Choosing an ansatz is the most crucial part in solving the above equation.**

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- **To find the solution up to $\mathcal{O}(\beta)$, we choose the ansatz as:**

$$\begin{aligned} ds^2 &= -r^{-2a}(1 - r^{c(\beta,r)})dt^2 + \frac{dr^2}{r^{-2a}(1 - r^{c(\beta,r)})r^4} + r^{-2a}d\vec{x}^2 \\ \Phi(r) &= m\mathrm{Log}(r) + \beta\Phi_1(r) \end{aligned} \tag{9}$$

where $c[\beta, r]$ is chosen as:

$$c[r] = c + \frac{Log(1 - \beta\kappa(r))}{Log(r)} \tag{10}$$

The first order correction $\kappa(r)$, is obtained as:

$$-(D-4)\left(4+\frac{(D-4)(D-1)16^{D-\frac{6\left((-1)^D+1\right)m}{D-2}}(g\times r)^{-aD+1}}{(D-2)\left((D-2)^2\alpha^2-16(D-1)\right)}\right)+\frac{2\Lambda r^{\frac{m}{2}(\alpha+\gamma)}}{(D-2)}$$

$$\left(r^{m\gamma+a(D-1)}\frac{\left((D-2)^2\alpha\left(3(D-2)\alpha+4(D-3)\gamma\right)-16(D-4)(D-3)\right)}{\left((D-2)^2\alpha\gamma+16(D-1)\right)}\right.$$

$$\left.-r^{m\alpha}\frac{\left(4(D-1)(D-2)^2\alpha^2+8(D-3)(D-2)^2\alpha\gamma+64(D-4)\right)}{\left((D-2)^2\alpha(\alpha+2\gamma)+16(D-1)\right)}\right),$$

Horizon gets a linear order correction as: $\mathbf{r}_h = 1+\beta r_1$.

Shear viscosity

Kubo formula for viscosity Low energy and low momentum limit of retarded Green's function of stress tensor in CFT.

$$\eta = -\lim_{\omega \to 0} \frac{1}{\omega} Im G^{R}_{xy;xy}(\omega, \mathbf{k} = 0) \tag{11}$$

- Translate the calculation of the correlator to a holographic one

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- Translate the calculation of the correlator to a holographic one
- perturbation in the xy components of the metric.

$$g_{xy} = g^0_{xy} + \epsilon h_{xy}(r, x) = g^0_{xy} + \epsilon g_{ii} \phi(r, x), \qquad \left[x = (t, \overrightarrow{x})\right]$$

with

$$\phi(r, x) = \frac{1}{(2\pi)^4} \int d^4 k e^{-ik.x} \phi(r, k),$$

$$\left[k = (\omega, \overrightarrow{k}), \qquad k.x = k_\mu x^\mu\right]$$

Entropy density

Wald formula

$$S = -2\pi \int_{\Sigma} d^{D-2}x\sqrt{-h}\frac{\delta L}{\delta R_{\mu\nu\rho\sigma}}\epsilon_{\mu\nu}\epsilon_{\rho\sigma} \quad (12)$$

1. $\epsilon_{\mu\nu}\epsilon^{\mu\nu} = -2$
2. $\epsilon_{\mu\nu} = \xi_\mu\eta_\nu - \xi_\nu\eta_\mu$

The L above is chosen by writing the action as

$$S_{action} \sim \int d^5x\sqrt{-g}L$$

Backreaction effects on η and s

For the matter coupled higher derivative action the shear viscosity and entropy density is

$$\eta = \frac{1}{2\kappa_D^2}\left[r_h^P - \frac{4\beta(D-2)^2\alpha\gamma\left(\alpha^2(D-2)^2 - 16(D-1)\right) r_0^M}{\left(16+\alpha^2(D-2)^2\right)^2}\right] \quad (13)$$

$$s = \frac{1}{2\kappa_D^2}\left[4\pi r_h^P - \frac{128\pi\beta(D-4)\left(\alpha^2(D-2)^2 - 16(D-1)\right) r_0^N}{\left(16+\alpha^2(D-2)^2\right)^2}\right] \quad (14)$$

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Giving,

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \frac{4\beta\left(-16(D-1)+(D-2)^2\alpha^2\right)\left(-8(D-4)+(D-2)^2\alpha\gamma\right)}{\left(16+(D-2)^2\alpha^2\right)^2}\right] \quad (15)$$

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Puzzle: **Backreaction to the matter solution still remains unknown**

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Thank You

Anti de Sitter space

- Solution to Einstein equation with negative cosmological constant.
- Space of Lorentzian signature $(-,+,+..+)$, but of constant negative curvature
- On boundary

$$-x_0^2 + \sum_{i=1}^{d-1} x_i^2 - x_{d+1}^2 = R^2$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \frac{1}{2}g_{\mu\nu}\Lambda = 0 \quad (16)$$

$$\implies R = \Lambda\frac{(d+1)}{d-1} \quad (17)$$

Thus constant negative curvature

$$R_{\mu\nu} = \frac{1}{d-1}\Lambda g_{\mu\nu} \quad (18)$$

Parametrize,

$$\Lambda = -\frac{d(d-1)}{L^2}$$

$$\implies R = -\frac{d(d+1)}{L^2} \quad (19)$$

Back

Conformal Field Theory

- Guage field theory with enhanced symmetries.
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- Guage field theory with enhanced symmetries.
- Poincare symmetry + dilatations + special conformal symmetry
- Dilatations: $x_\mu \to \lambda x_\mu$
- special conformal: Translation+Inversion+Translation

Under special conformal: $x'^\mu = x^\mu + 2x^\mu b.x - x^2 b^\mu$

Back

$$\begin{aligned} \text{Translation:} P_\mu &= -i\partial_\mu & (20) \\ \text{Dilatations:} D &= -ix^\mu \partial_\mu & (21) \\ \text{Rotations:} L_{\mu\nu} &= i(x_\mu \partial_\nu - x_\nu \partial_\mu) & (22) \\ \text{SCT:} G_\mu &= -2ix_\mu (x.\partial) + ix^2 \partial_\mu & (23) \end{aligned}$$

Back